Al-Mustaqbal University
College of Engineering Technology
Department of Cyber Security Techniques Engineering
Class: 1st
Subject: Math
Lecturer: Dr. Hussein Ali Ameen
2nd term – Lecture: 1 – Implicit and Partial Differentiation

Implicit Differentiation:

Most of the functions we have dealt with so far have been described by an equation of the form $y=f(x)$ that expresses y explicitly in terms of the variable x. We have learned rules for differentiating functions defined in this way. Another situation occurs when we encounter equations like

$$y^2 - x^2 = 0, y^3 + 8x = 3, \qquad yx + 4y - 6 = 0$$

Definition:

1. Differentiate both sides of the equation with respect to x, treating y as a differentiable function of x.

2. Collect the terms with dy / dx on one side of the equation and solve for dy / dx.

Example: Find $\frac{dy}{dx}$ for the equation $y^2 + x^3 - 9xy = 0$

Sol:

$$2y\frac{dy}{dx} + 3x^2 - \left(9x\frac{dy}{dx} + 9y\right) = 0 \rightarrow 2y\frac{dy}{dx} + 3x^2 - 9x\frac{dy}{dx} - 9y = 0$$

$$2y\frac{dy}{dx} - 9x\frac{dy}{dx} = 9y - 3x^2 \rightarrow \frac{dy}{dx}(2y - 9x) = 9y - 3x^2$$

$$\frac{dy}{dx} = \frac{9y-3x^2}{2y-9x}$$

We can refer to $\frac{dy}{dx} = as\ a\ y'$ and $\frac{d^2y}{dx^2} = as\ a\ y''etc..$ وذلك لسهولة الحل

هل من الممكن اعادة المثال اعلاه باستخدام طريقة الاختصار أعلاه؟ حاول؟

hussein_awadh@uomus.edu.iq

Example: Use implicit differentiation to find $dy/\ dx$ for the equations

1- $y^2 = \frac{x-1}{x+1}$

2- $x cos\,(2x + 3\,) = y \sin x$

Sol: 1- $y^2 = \frac{x-1}{x+1}$

$$2y\frac{dy}{dx} = \frac{(x+1).1-(x-1).1}{(x-1)^2} = \frac{x+1-x+1}{(x-1)^2} = \frac{2}{(x-1)^2}$$

$$\frac{dy}{dx} = \frac{2}{2y(x-1)^2} = \frac{1}{y(x-1)^2}$$

2- $x cos\,(2x + 3) = y \sin x$

Sol:

$$-x\sin(2x+3)\,.\,2 + \cos\ (2x+3)\,.\,1 = y\cos x + \frac{dy}{dx}\sin x$$

$$-2x\sin(2x+3) + \cos(2x+3) = y\cos x + \frac{dy}{dx}\sin x$$

$$-2x\sin(2x+3) + \cos(2x+3) - y\cos x = \frac{dy}{dx}\sin x$$

$$\frac{dy}{dx} = \frac{-2x\sin(2x+3) + \cos(2x+3) - y\cos x}{\sin x}$$

Example: find dy/dx for the equation $y^2 = X^2 + \sin xy$

$$\frac{d}{dx}(y^2) = \frac{d}{dx}(x^2) + \frac{d}{dx}(\sin xy)$$

$$2y\frac{dy}{dx} = 2x + (\cos xy)\left(x\frac{dy}{dx} + y\right)$$

$$2y\frac{dy}{dx} - (\cos xy)\left(x\frac{dy}{dx} + y\right) = 2x$$

$$2y\frac{dy}{dx} - (\cos xy)\left(x\frac{dy}{dx}\right) = 2x + (\cos xy)y$$

$$\big(2y - x(\cos xy)\big)\frac{dy}{dx} = 2x + y(\cos xy)$$

$$\frac{dy}{dx} = \frac{2x + y(\cos xy)}{2y - x(\cos xy)}$$

Tips for Successfully Tackling Implicit Differentiation Problems

The key to mastering implicit differentiation lies in practice and understanding the underlying rules. Here are some helpful tips as you work through more problems:

- Always apply the chain rule: Remember when differentiating y terms, multiply by dy/dx.
- Use the product rule carefully: When differentiating products like xy, treat y as a function of x.
- Keep dy/dx terms on one side: After differentiating, isolate dy/dx to solve for it easily.

- Substitute known values: In some problems, you might need to find the slope at a particular point. Substitute x and y values after finding dy/dx.
- Practice with diverse equations: The more varied implicit differentiation practice problems you solve, the more intuitive the process becomes.

Home Work

find d^2y/dx^2 for the equation $2x^3 - 3y^2 = 8$

Find the derivative of the following function:

$$x^2y^2 = x^2 + y^2$$

Trigonometric function:

1- If $y = f(x) = \sin x$ *then* $\frac{dy}{dx} = \cos x$

2- If $y = f(x) = \cos x$ *then* $\frac{dy}{dx} = -\sin x$

3- If $y = f(x) = \tan x$ *then* $\frac{dy}{dx} = \sec^2 x$

4- If $y = f(x) = \cot x$ *then* $\frac{dy}{dx} = -\csc^2 x$

5- If $y = f(x) = \sec x$ *then* $\frac{dy}{dx} = \sec x \tan x$

6- If $y = f(x) = \csc x$ *then* $\frac{dy}{dx} = -\csc x \cot x$

Derivative of the inverse trigonometric function:

$$(1)\ y = \sin^{-1} x \ \rightarrow \acute{y} = \frac{1}{\sqrt{1-x^2}}$$

$$(2)\ y = \cos^{-1} x \ \rightarrow \acute{y} = \frac{-1}{\sqrt{1-x^2}}$$

$$(3)\ y = \tan^{-1} x \ \rightarrow \acute{y} = \frac{1}{1+x^2}$$

$$(4)\ y = \cot^{-1} x \ \rightarrow \acute{y} = \frac{-1}{1+x^2}$$

$$(5)\ y = \sec^{-1} x \ \rightarrow \acute{y} = \frac{1}{x\sqrt{x^2-1}}$$

$$(6)\ y = \csc^{-1} x \ \rightarrow \acute{y} = \frac{-1}{x\sqrt{x^2-1}}$$

Derivative of the natural logarithm:

$$If \ \ y = f(x) = \ln x \ \ then: \qquad \frac{dy}{dx} = \acute{f}(x) = \frac{1}{x}$$

Example: Find $\frac{dy}{dx}$ of the following functions:

1- $y = \ln(x^3 + 2x^2 - 1)$

Sol:

$$\frac{dy}{dx} = \frac{1}{x^3+2x^2-1} \cdot (3x^2 + 4x)$$