Derivative of the inverse trigonometric function:

$$(1)\ y = \sin^{-1} x \ \rightarrow \acute{y} = \frac{1}{\sqrt{1-x^2}}$$

$$(2)\ y = \cos^{-1} x \ \rightarrow \acute{y} = \frac{-1}{\sqrt{1-x^2}}$$

$$(3)\ y = \tan^{-1} x \ \rightarrow \acute{y} = \frac{1}{1+x^2}$$

$$(4)\ y = \cot^{-1} x \ \rightarrow \acute{y} = \frac{-1}{1+x^2}$$

$$(5)\ y = \sec^{-1} x \ \rightarrow \acute{y} = \frac{1}{x\sqrt{x^2-1}}$$

$$(6)\ y = \csc^{-1} x \ \rightarrow \acute{y} = \frac{-1}{x\sqrt{x^2-1}}$$

Derivative of the natural logarithm:

$$If \ \ y = f(x) = \ln x \ \ then: \qquad \frac{dy}{dx} = \acute{f}(x) = \frac{1}{x}$$

Example: Find $\frac{dy}{dx}$ of the following functions:

1- $y = \ln(x^3 + 2x^2 - 1)$

Sol:

$$\frac{dy}{dx} = \frac{1}{x^3+2x^2-1} \,.\, (3x^2 + 4x)$$

2- $y = \ln(x^{-2} + \sin^2 3x)$

Sol:

$$\frac{dy}{dx} = \frac{1}{x^{-2}+\sin^2 3x} \,.\, -2x^{-3} + 2\sin(3x)\cos(3x)\,.\,3$$

3- $y = \sin^{-1}(\ln x)\,.\ln(sin^{-1}\ 3x)$

Sol:

$$\frac{dy}{dx} = \sin^{-1}(\ln x).\frac{1}{\sin^{-1} 3x}.\frac{3}{\sqrt{1-(3x)^2}} + \ln(\sin^{-1} 3x).\frac{1}{\sqrt{1-(\ln x)^2}}\,.\frac{1}{x}$$

4- $y = \ln[\ln(\sec^2 2x + x\sin^{-1} x)]$

Sol:

$$\frac{dy}{dx} = \frac{1}{\ln(\sec^2 2x + x\sin^{-1} x)}\,.\frac{1}{\sec^2 2x + x\sin^{-1} x}\,.2\sec(2x)\,(\sec(2x)\tan(2x).2)$$
$$+\frac{x}{\sqrt{1-x^2}} + \sin^{-1} x$$

Example: Find $\frac{dy}{dx}$ for the following:

1) $y = x^{\cos x}$

Sol: Take ln for both sides

$$y = x^{\cos x} \quad \rightarrow \ln y = ln x^{\cos x}$$

$$\ln y = \cos x \ln x$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{\cos x}{x} - \sin x \ln x$$

$$\frac{dy}{dx} = y\left[\frac{\cos x}{x} - \sin x \ln x\right] = x^{\cos x}\left[\frac{\cos x}{x} - \sin x \ln x\right]$$

2) $y = (\ln x)^x$

Sol:

$$\ln y = \ln\left[(\ln x)^x\right] = \ln\left[x \ln x\right]$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x \ln x} \cdot \left(x . \frac{1}{x} + \ln x\right) = \frac{\ln x + 1}{x \ln x}$$

$$\therefore \frac{dy}{dx} = y\left[\frac{\ln x + 1}{x \ln x}\right] = (\ln x)^x\left[\frac{\ln x + 1}{x \ln x}\right]$$

Example: Solve for x if $2^x = 4^{x-1}$

Take ln for both sides

$$\ln(2^x) = \ln(4^{x-1})$$

$$x \ln 2 = (x-1)\ln 4 = x\ln 4 - \ln 4$$

$$x\, ln\, 2 - x \ln 4 = -\ln 4$$

$$x\,(\ln 2 - \ln 4) = -\ln 4 \quad \rightarrow \quad \therefore x = \frac{-\ln 4}{\ln 2 - \ln 4}$$

Derivative of the exponential function

$$\text{If } \boldsymbol{y = e^x} \;\; \boldsymbol{then} \;\; \frac{\boldsymbol{dy}}{\boldsymbol{dx}} = \boldsymbol{e^x}$$

Now, if $u = u(x)$ $then\ y = e^u$

$$\frac{\boldsymbol{du}}{\boldsymbol{dx}} = \boldsymbol{e^u}.\frac{\boldsymbol{du}}{\boldsymbol{dx}}$$

Example: Find $\frac{dy}{dx}$ of the following functions.

1- $y = e^{x^2+\sin 2x}$

$$\frac{dy}{dx} = e^{x^2+\sin 2x}\,.(2x + 2\cos 2x)$$

2- $y = e^{(\tan^{-1} 2x)+\ln x}$

$$\frac{dy}{dx} = e^{(\tan^{-1} 2x)+\ln x}.\left(\frac{2x}{1+4x^2} + \frac{1}{x}\right)$$

3- $y = \tan^{-1}(e^{2x})$

$$\frac{dy}{dx} = \frac{1}{1+e^{4x^2}} \, . e^{2x}. 2 = \frac{2\, e^{2x}}{1+e^{4x^2}}$$

4- $y = e^{\sec x}. \sec e^{x}$

$$\frac{dy}{dx} = e^{\sec x}. (\sec e^{x} \tan e^{x}. e^{x}) + \sec e^{x}. e^{\sec x}. (\sec x \, tan \, x)$$

Application of Differentiation

a- Partial Derivatives:

The partial derivative of $f(x,y)$ with respect to x at the point $(x°,y°)$ is:

$$\left.\frac{\partial f}{\partial x}\right|_{(x_\circ,y_\circ)} = \frac{d}{dx} f(x, y_\circ) = f_x$$

The partial derivative of $f(x,y)$ with respect to y at the point $(x°,y°)$ is:

$$\left.\frac{\partial f}{\partial y}\right|_{(x_\circ,y_\circ)} = \frac{d}{dy} f(x_\circ, y) = f_y$$

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Example 1: Find the values of $\frac{\partial f}{\partial x}$, and $\frac{\partial f}{\partial y}$ *at the point* (2,−1) *if*

$$f(x,y) = x^2 + 3xy + y - 1$$

Solution:

To find $\frac{\partial f}{\partial x}$, we treat y as a constant and differentiate with respect to x,

$$\frac{\partial f}{\partial x} = 2x + 3y + 0 - 0 = 2x + 3y$$

The values of $\frac{\partial f}{\partial x}$ at the point (2,-1) is: $2 * 2 + 3 * -1 = 1$

To find $\frac{\partial f}{\partial y}$, we treat x as a constant and differentiate with respect to y,

$$\frac{\partial f}{\partial y} = 0 + 3x + 1 - 0 = 3x + 1$$

The values of $\frac{\partial f}{\partial y}$ at the point (2,-1) is: $3 * 2 + 1 = 7$

Example 2: Find the values of $\frac{\partial f}{\partial y}$ *if* $f(x,y)=y\sin(xy)$

Solution:

$$\frac{\partial f}{\partial y} = y * \cos(xy) * x + \sin(xy) * 1 = xy\,cos(xy) + \sin(xy)$$

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Example 3: Find the values of $\frac{\partial f}{\partial x}$, and $\frac{\partial f}{\partial y}$ $if\ f(x,y)=\frac{2y}{y+cos\ x}$

Solution:

$$\frac{\partial f}{\partial x}=\frac{(y+\cos x)*0-[2y(0-\sin x)]}{(y+\cos x)^{\ 2}}=\frac{2y\sin x}{(y+\cos x)^{\ 2}}$$

$$\frac{\partial f}{\partial y}=\frac{(y+\cos x)*2-[2y(1+0)]}{(y+\cos x)^{\ 2}}=\frac{2(y+\cos x)-2y}{(y+\cos x)^{\ 2}}$$

Example 4: Find the values of $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$ and $\frac{\partial f}{\partial z}$ $if\ f(x,y,z)=\sqrt{x^2+y^2+z^2}$

$$\left(\frac{\partial f}{\partial x}\right)=f_x=\frac{1}{2}(x^2+y^2+z^2)^{\frac{-1}{2}}*(2x)=\frac{x}{\sqrt{x^2+y^2+z^2}}$$

$$\left(\frac{\partial f}{\partial y}\right)=f_y=\frac{1}{2}(x^2+y^2+z^2)^{\frac{-1}{2}}*(2y)=\frac{y}{\sqrt{x^2+y^2+z^2}}$$

$$\left(\frac{\partial f}{\partial z}\right)=f_z=\frac{1}{2}(x^2+y^2+z^2)^{\frac{-1}{2}}*(2z)=\frac{z}{\sqrt{x^2+y^2+z^2}}$$

Apply the partial derivatives for the following functions

- $\frac{\partial f}{\partial x}$, and $\frac{\partial f}{\partial y}$ $f(x,y) = \ln\sqrt{\boldsymbol{x^2} + \boldsymbol{y^2}}$
- $f(x,y) = e^{x}cosy$
- $f(x,y) = e^{3x}cosy$
- $f(x,y) = e^{3x^2}cosy$
- $f(x,y,z,w) = x^3e^{3y-4z}cos2w$