Al-Mustaqbal University
College of Engineering Technology
Department of Cyber Security Techniques Engineering
Class: 1st
Subject: Math
Lecturer: Dr. Hussein Ali Ameen
1st term – Lecture: 1+2 - Introduction to Matrices and Grammar's Rule

Definition

A matrix is a set of real or complex numbers (or elements) arranged in rows and columns to form rectangular array. A matrix having m rows and n columns is called ($m\times \boldsymbol{n}$) matrix and is referred to as having order ($m\times \boldsymbol{n}$) . A matrix is in dictating by writing the array within brackets.

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

A horizontal line of elements is called row, and a vertical line is called a column.

For example:

$$\begin{bmatrix} 5 & 7 & 2 \\ 6 & 3 & 8 \end{bmatrix} \text{ is } (2\times 3) \text{ matrix}$$

2 the numbers of rows and 3 the number of columns.

(Discussion)

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Types of Matrices:

-Row matrix: consists of 1 row only, for example [4 3 7 2] is a row matrix of -order (1× 4).

-Column matrix: consists of 1 column only, for example

$$\begin{bmatrix} 6 \\ 3 \\ 8 \end{bmatrix} \text{ is a column matrix of order } (3\times 1).$$

-Square matrix: is a matrix in which the number of rows (**m**) equals the number of columns (**n**) for example

$$S = \begin{bmatrix} 2 & 0 & 5 \\ 7 & 8 & 7 \\ 6 & 7 & 5 \end{bmatrix}$$

-**Rectangular matrix:** A matrix of any size **(m×** $\boldsymbol{n}$**)** and this includes square matrices as a special case.

- Diagonal matrix: is a square matrix with all elements zero except those on the main diagonal. For example

$$D = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 7 \end{bmatrix}$$

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-**Unit matrix:** is a diagonal matrix in which the elements on the main diagonal are all unity. For example

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

-**Null (zero) matrix:** is one whose elements are zero. For example

$$O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

-**Vector matrix:** is a matrix with only one row or column. Its entries are called the component of the vector.

Some Operations on Matrices:

1- Equality of matrices:

Tow matrices A and B are equal if and only if they have the same size and corresponding entries are equal, matrices that are not equal are called different.

Example: let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ $and\ B = \begin{bmatrix} 4 & 0 \\ 3 & -1 \end{bmatrix}$

A=B if and only if: $a_{11} = 4, a_{12} = 0, a_{21} = 3, a_{22} = -1$

Example: If $A = \begin{bmatrix} 2 & 3 & a \\ b & 6 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 & 9 \\ -3 & 6 & 1 \end{bmatrix}$ $and\ C = \begin{bmatrix} 2 & 3 & 9 \\ -3 & 6 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

Then $A = B\ if\ a = 9\ and\ b = -3\ but\ \ A \neq C\ and\ B \neq C.$

2- Addition and Subtraction of Matrices:

To be added or subtracted, two matrices must be of the same order. The sum or difference is then determined by adding or subtracting corresponding elements. For example:

$$\begin{bmatrix}4 & 2 & 3\\5 & 7 & 6\end{bmatrix}+\begin{bmatrix}1 & 8 & 9\\3 & 5 & 4\end{bmatrix}=\begin{bmatrix}4+1 & 2+8 & 3+9\\5+3 & 7+5 & 6+4\end{bmatrix}=\begin{bmatrix}5 & 10 & 12\\8 & 12 & 10\end{bmatrix}$$

$$\begin{bmatrix}6 & 5 & 12\\9 & 4 & 8\end{bmatrix}-\begin{bmatrix}3 & 7 & 1\\2 & 10 & -5\end{bmatrix}=\begin{bmatrix}6-3 & 5-7 & 12-1\\9-2 & 4-10 & 8+5\end{bmatrix}=\begin{bmatrix}3 & -2 & 11\\7 & -6 & 13\end{bmatrix}$$

3- Multiplication of Matrices:

Tow matrices can be multiplied together only when the number of columns in the first is equal to the number of rows in the second, for example:

If $A=\begin{bmatrix}a_{11} & a_{12} & a_{13}\\a_{21} & a_{22} & a_{23}\end{bmatrix}$ $and\ B=\begin{bmatrix}b_1\\b_2\\b_3\end{bmatrix}$

Then:

$$A.B=\begin{bmatrix}a_{11} & a_{12} & a_{13}\\a_{21} & a_{22} & a_{23}\end{bmatrix}\begin{bmatrix}b_1\\b_2\\b_3\end{bmatrix}=\begin{bmatrix}a_{11}b_1+ & a_{12}b_2 & +a_{13}b_3\\a_{21}b_1+ & a_{22}b_2 & +a_{23}b_3\end{bmatrix}$$

Example: If $A = \begin{bmatrix} 4 & 7 & 6 \\ 2 & 3 & 1 \end{bmatrix}$ $and\ B = \begin{bmatrix} 8 \\ 5 \\ 9 \end{bmatrix}, find\ A.B$

Sol:

$$A.B = \begin{bmatrix} 4 & 7 & 6 \\ 2 & 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 8 \\ 5 \\ 9 \end{bmatrix} = \begin{bmatrix} 4*8+7*5+6*9 \\ 2*8+3*5+1*9 \end{bmatrix} = \begin{bmatrix} 32+35+54 \\ 16+15+9 \end{bmatrix} = \begin{bmatrix} 121 \\ 40 \end{bmatrix}$$

Example: If $A = \begin{bmatrix} 1 & 5 \\ 2 & 7 \\ 3 & 4 \end{bmatrix}$ $and\ B = \begin{bmatrix} 8 & 4 & 3 & 1 \\ 2 & 5 & 8 & 6 \end{bmatrix}, find\ A.B$

Sol:

$$A.B = \begin{bmatrix} 1 & 5 \\ 2 & 7 \\ 3 & 4 \end{bmatrix} . \begin{bmatrix} 8 & 4 & 3 & 1 \\ 2 & 5 & 8 & 6 \end{bmatrix} = \begin{bmatrix} 8+10 & 4+25 & 3+40 & 1+30 \\ 16+14 & 8+35 & 6+56 & 2+42 \\ 24+8 & 12+20 & 9+32 & 3+24 \end{bmatrix}$$

$$= \begin{bmatrix} 18 & 29 & 43 & 31 \\ 30 & 43 & 62 & 44 \\ 32 & 32 & 41 & 27 \end{bmatrix}$$

Note: If A is an **(m×** $\boldsymbol{n}$**)** matrix and B is **(n×** $\boldsymbol{m}$**)** matrix, then products **A.B** and **B.A** are possible.

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Example: If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}, B = \begin{bmatrix} 7 & 10 \\ 8 & 11 \\ 9 & 12 \end{bmatrix}, find\ A.B\ and\ B.A$

Sol:

$$A.B = \begin{bmatrix} 7+16+27 & 10+22+36 \\ 28+40+54 & 40+55+72 \end{bmatrix} = \begin{bmatrix} 50 & 68 \\ 122 & 167 \end{bmatrix}$$

$$B.A = \begin{bmatrix} 7 & 10 \\ 8 & 11 \\ 9 & 12 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} 7+40 & 14+50 & 21+60 \\ 8+44 & 16+55 & 24+66 \\ 9+48 & 18+60 & 27+72 \end{bmatrix}$$

$$= \begin{bmatrix} 47 & 64 & 81 \\ 52 & 71 & 90 \\ 57 & 78 & 99 \end{bmatrix}$$

Note: $\boldsymbol{A.B \neq B.A}$ Multiplication is not commutative.

Properties of Matrix Operations:

If $A\ and\ B, C$ are $(\mathbf{m} \times \boldsymbol{n})$ matrices, O is zero matrix and K, R are any scalars then:

- $A + B = B + A$
- $(A + B) + C = A + (B + C)$
- $A + 0 = A,\ A + (-A) = 0$
- $K\ (RA) = (K\ R)A, (K + R)A = K\ A + RA$
- $I\ A = A\ ,\ 0\ A = 0\ ,\ R\ 0 = 0$

Transpose of Matrix:

If the rows and columns of matrix are interchanged, then the new matrix is called the transpose of the original matrix. If $\boldsymbol{A^T}$ is the transpose of matrix $\boldsymbol{A}$**,** then

$\boldsymbol{A \neq A^T}$ **,** For example**:**

$$A = \begin{bmatrix} 4 & 6 \\ 7 & 9 \\ 2 & 5 \end{bmatrix} \rightarrow A^T = \begin{bmatrix} 4 & 7 & 2 \\ 6 & 9 & 5 \end{bmatrix}$$

Example: If $A = \begin{bmatrix} 2 & 7 & 6 \\ 3 & 1 & 5 \end{bmatrix}$ $and\ B = \begin{bmatrix} 4 & 0 \\ 3 & 7 \\ 1 & 5 \end{bmatrix}, find\ A.B\ and\ (A.B)^T.$

Sol:

$$A.B = \begin{bmatrix} 35 & 79 \\ 20 & 32 \end{bmatrix} \quad , \quad (A.B)^T = \begin{bmatrix} 35 & 20 \\ 79 & 32 \end{bmatrix}$$

Special Matrices:

Square matrix is a matrix of order $(\boldsymbol{m \times m})$**.** A square matrix is **symmetric** if $a_{ij} = a_{ji}$ $means\ \boldsymbol{A} = \boldsymbol{A^T}\ \ for\ example$

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 8 & 9 \\ 5 & 9 & 4 \end{bmatrix}, \quad A^T = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 8 & 9 \\ 5 & 9 & 4 \end{bmatrix}$$

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A square matrix is **skew–symmetric** if $a_{ij} = -a_{ji}$ *means* $\boldsymbol{A} \neq \boldsymbol{A}^{\boldsymbol{T}}$ *for example*

$$A = \begin{bmatrix} 0 & 2 & 5 \\ -2 & 0 & 9 \\ -5 & -9 & 0 \end{bmatrix}, \qquad A^T = \begin{bmatrix} 0 & -2 & -5 \\ 2 & 0 & -9 \\ 5 & 9 & 0 \end{bmatrix}$$

Example: Given that $A = \begin{bmatrix} 4 & 2 & 6 \\ 1 & 8 & 7 \end{bmatrix}$ determine A^T *and* $A.A^T$.

Sol:

$$A = \begin{bmatrix} 4 & 2 & 6 \\ 1 & 8 & 7 \end{bmatrix} \qquad , A^T = \begin{bmatrix} 4 & 1 \\ 2 & 8 \\ 6 & 7 \end{bmatrix}$$

$$A.A^T = \begin{bmatrix} 4 & 2 & 6 \\ 1 & 8 & 7 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ 2 & 8 \\ 6 & 7 \end{bmatrix} = \begin{bmatrix} 16+4+36 & 4+16+42 \\ 4+16+42 & 4+64+49 \end{bmatrix} = \begin{bmatrix} 56 & 62 \\ 62 & 117 \end{bmatrix}$$

Determinant of a Square Matrix:

The determinant of a square matrix is the determinant having the same elements as those of the matrix. For example

$A = \begin{bmatrix} 5 & 2 & 1 \\ 0 & 6 & 3 \\ 8 & 4 & 7 \end{bmatrix}$ then the det of A is given by:

$$|A| = 5(42-12) - 2(0-24) + 1(0-48) = 5(30) - 2(-24) + 1(-48)$$
$$= 150 + 48 - 48 = 150$$

Note that the transpose of $A^T = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 6 & 4 \\ 1 & 3 & 7 \end{bmatrix}$

And the determinate of A^T is

$$|A^T| = 5(42 - 12) - 0(14 - 4) + 8(6 - 6) = 5(30) = 150$$

Note that: $\boldsymbol{det}\,(\boldsymbol{A}) = \boldsymbol{det}\,(\boldsymbol{A^T})$.

Cofactors:

If A is square matrix, the determinates of its element will be:

If A is square matrix, the determinates of its element will be:

$$A = \begin{bmatrix} 2 & 3 & 5 \\ 4 & 1 & 6 \\ 1 & 4 & 0 \end{bmatrix}$$

$$\det(A) = 2(0 - 24) - 3(0 - 6) + 5(16 - 1) = 45$$

The minor of element 2 is $+\begin{bmatrix} 1 & 6 \\ 4 & 0 \end{bmatrix} = 0 - 24 = -24$

Similarly the cofactor of element 3 is $-\begin{bmatrix} 4 & 6 \\ 1 & 0 \end{bmatrix} = -(0 - 6) = 6$

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The cofactor of element 5 is $+\begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix} = +(16-1) = 1$

The cofactor of element 4 is $-\begin{bmatrix} 3 & 5 \\ 4 & 0 \end{bmatrix} = -(0-20) = 20$

The cofactor of element 1 is $+\begin{bmatrix} 2 & 5 \\ 1 & 0 \end{bmatrix} = +(0-5) = -5$

The cofactor of element 6 is $-\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} = -(8-3) = -5$

The cofactor of element 1 is $+\begin{bmatrix} 3 & 5 \\ 1 & 6 \end{bmatrix} = +(18-5) = 13$

The cofactor of element 4 is $-\begin{bmatrix} 2 & 5 \\ 4 & 6 \end{bmatrix} = -(12-20) = 8$

The cofactor of element 0 is $+\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix} = +(2-12) = -10$

The cofactor matrix C is

$$C = \begin{bmatrix} -24 & 6 & 15 \\ 20 & -5 & -5 \\ 13 & 8 & -10 \end{bmatrix}$$

And the transpose of C is

$$C^T = \begin{bmatrix} -24 & 20 & 13 \\ 6 & -5 & 8 \\ 15 & -5 & -10 \end{bmatrix}$$

Where C^T is called the adjoint of matrix $A = adj\ A$.

Inverse of a Square Matrix:

If $A = \begin{bmatrix} 2 & 3 & 5 \\ 4 & 1 & 6 \\ 1 & 4 & 0 \end{bmatrix}$ *then the inverse of A is* A^{-1} *and given by*:

$$A^{-1} = \frac{adj\ A}{\det A}$$

$$\det A = |A| = 2(0-24) - 3(0-6) + 5(16-1) = 45$$

The cofactor matrix C is:

$$C = \begin{bmatrix} -24 & 6 & 15 \\ 20 & -5 & -5 \\ 13 & 8 & -10 \end{bmatrix} \text{ and } C^T = adj\ A \begin{bmatrix} -24 & 20 & 13 \\ 6 & -5 & 8 \\ 15 & -5 & -10 \end{bmatrix}$$

$$A^{-1} = \frac{adj\ A}{\det A} = \frac{1}{45}\begin{bmatrix} -24 & 20 & 13 \\ 6 & -5 & 8 \\ 15 & -5 & -10 \end{bmatrix} = \begin{bmatrix} -24/45 & 20/45 & 13/45 \\ 6/45 & -5/45 & 8/45 \\ 15/45 & -5/45 & -10/45 \end{bmatrix}$$

Example: Find the inverse of the given matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 1 & 5 \\ 6 & 0 & 2 \end{bmatrix}$$

Sol:

$\det A = |A| = 1(2-0) - 2(8-30) + 3(0-6) = 28$

The cofactor of element 1 is $+(2-0) = 2$

The cofactor of element 4 is $-(4-0) = -4$

The cofactor of element 6 is $+(10-3) = 7$

The cofactor of element 2 is $-(8-30) = 22$

The cofactor of element 1 is $+(2-18) = -16$

The cofactor of element 0 is $-(5-12) = 7$

The cofactor of element 3 is $+(0-6) = -6$

The cofactor of element 5 is $-(0-12) = 12$

The cofactor of element 2 is $+(1-8) = -7$

The cofactor matrix C is

$$C = \begin{bmatrix} 2 & 22 & -6 \\ -4 & -16 & 12 \\ 7 & 7 & -7 \end{bmatrix} \quad and \quad adj\ A = C^T = \begin{bmatrix} 2 & -4 & 7 \\ 22 & -16 & 7 \\ -6 & 12 & -7 \end{bmatrix}$$

$$C = \begin{bmatrix} 2 & 22 & -6 \\ -4 & -16 & 12 \\ 7 & 7 & -7 \end{bmatrix} \quad and \quad adj\ A = C^T = \begin{bmatrix} 2 & -4 & 7 \\ 22 & -16 & 7 \\ -6 & 12 & -7 \end{bmatrix}$$

$$A^{-1} = \frac{adj\ A}{\det A} = \frac{1}{28}\begin{bmatrix} 2 & -4 & 7 \\ 22 & -16 & 7 \\ -6 & 12 & -7 \end{bmatrix} = \begin{bmatrix} 2/28 & -4/28 & 7/28 \\ 22/28 & -16/28 & 7/28 \\ -6/28 & 12/28 & -7/28 \end{bmatrix}$$

$$= \begin{bmatrix} 1/14 & -1/7 & 1/4 \\ 11/14 & -8/14 & 1/4 \\ -3/14 & 6/14 & -1/4 \end{bmatrix}$$

HOMEWORK

Find $A.A^{-1}$ *if the matrix A is given by*: $A = \begin{bmatrix} 3 & 0 & 1 \\ 0 & 0 & 2 \\ -1 & 1 & 5 \end{bmatrix}$

If $A = \begin{bmatrix} x^2 & 2 & 9 \\ 1+y & 4 & 0 \\ 2 & 3 & 3 \end{bmatrix}$ ***and*** $B = \begin{bmatrix} 0 & 2 & 9 \\ 5 & 4 & 0 \\ 2 & 3 & 3 \end{bmatrix}$, **find the values of** x, y.

Grammar's rule for solving a set of linear equations:

Consider a set of linear equations in three unknowns x, y, z

$$a_{11}x + a_{12}y + a_{13}z = b_1 \qquad (1)$$

$$a_{21}x + a_{22}y + a_{23}z = b_2 \qquad (2)$$

$$a_{31}x + a_{32}y + a_{33}z = b_3 \qquad (3)$$

In matrices notation the system of linear equations may be written as:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

The above theorem called Grammar's rule to solve it we put:

$$D = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \qquad D_1 = \begin{bmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{bmatrix}$$

$$D_2 = \begin{bmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{bmatrix}, \qquad D_3 = \begin{bmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{bmatrix}$$

If $D \neq 0$ then the system has unique solution.

$$\therefore x = \frac{D_1}{D}, \qquad y = \frac{D_2}{D}, \qquad z = \frac{D_3}{D}$$

Example: Use Grammar's rule to solve the system

$5x - 2y = -1$

$2x + 3y = 3$

$$\begin{bmatrix} 5 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

$$D = \begin{bmatrix} 5 & -2 \\ 2 & 3 \end{bmatrix} = 15 + 4 = 19$$

$$D_1 = \begin{bmatrix} -1 & -2 \\ 3 & 3 \end{bmatrix} = -3 + 6 = 3$$

$$D_2 = \begin{bmatrix} 5 & -1 \\ 2 & 3 \end{bmatrix} = 15 + 2 = 17$$

$$\therefore x = \frac{D_1}{D} = \frac{3}{19} \quad , \quad y = \frac{D_2}{D} = \frac{17}{19}$$

Example: Use Grammar's rule to solve the system

$x + 2z = 6$

$-3x + 4y + 6z = 30$

$-x - 2y + 3z = 8$

Sol:

$$\begin{bmatrix} 1 & 0 & 2 \\ -3 & 4 & 6 \\ -1 & -2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 30 \\ 8 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & 0 & 2 \\ -3 & 4 & 6 \\ -1 & -2 & 3 \end{bmatrix} = 1(12+12) - 0 + 2\,(6+4) = 24 + 20 = 44$$

$$D1 = \begin{bmatrix} 6 & 0 & 2 \\ 30 & 4 & 6 \\ 8 & -2 & 3 \end{bmatrix} = 6(12 + 12) + 2(-60 - 32) = 144 - 184 = -40$$

$$D2 = \begin{bmatrix} 1 & 6 & 2 \\ -3 & 30 & 6 \\ -1 & 8 & 3 \end{bmatrix} = 1(90 - 48) - 6(-9 + 6) + 2(-24 + 30)$$

$$= 42 + 18 + 12 = 72$$

$$D3 = \begin{bmatrix} 1 & 0 & 6 \\ -3 & 4 & 30 \\ -1 & -2 & 8 \end{bmatrix} = 1(32 + 60) - 6(6 + 4) = 92 + 60 = 152$$

$$\therefore x = \frac{D_1}{D} = -\frac{40}{44} \quad , \qquad y = \frac{D_2}{D} = \frac{72}{44} \;, \qquad z = \frac{D_3}{D} = \frac{152}{44}$$

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Solve the following equations using Grammar's rule:

a) $3x + 8y = 4\ 3x$

$-\ y = -13$

b) $2x + y - z = 2$

$x - y + z = 7\ \ 2x +$

$2y + z = 4$

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