كلية التقنيات الهندسية

هندسة تقنيات الذكاء الاصطناعي

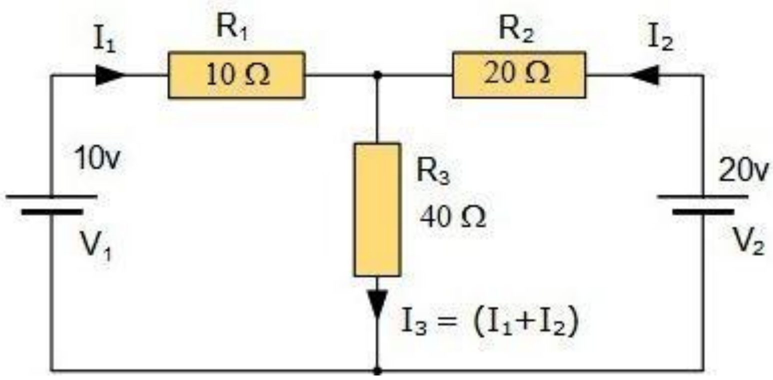
Lecture: (6)

Mesh Current Analysis

Introduction

While Kirchhoff´s Laws give us the basic method for analysing any complex electrical circuit, there are different ways of improving upon this method by using Mesh Current Analysis or Nodal Voltage Analysis that results in a lessening of the math's involved and when large networks are involved this reduction in maths can be a big advantage.

Mesh Current Analysis Circuit



One simple method of reducing the amount of math's involved is to analyze the circuit using Kirchhoff's Current Law equations to determine the currents, I1 and I2 flowing in the two resistors. Then there is no need to calculate the current I3 as its just the sum of I1 and I2. So Kirchhoff's second voltage law simply becomes:

Equation No 1 : $10 = 50I1 + 40I2$

Equation No 2 : $20 = 40I1 + 60I2$

Mesh Current Analysis

An easier method of solving the above circuit is by using Mesh Current Analysis or Loop Analysis which is also sometimes called Maxwell´s Circulating Currents method. Instead of labeling the branch currents we need to label each "closed loop" with a circulating current.

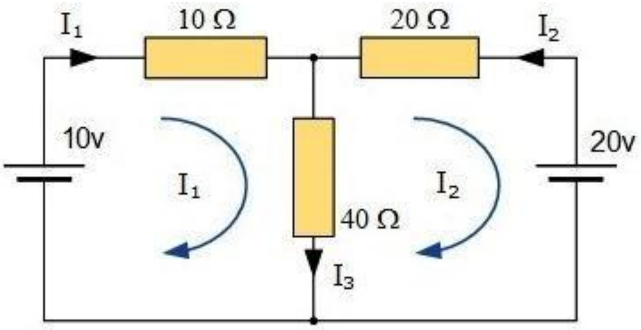
As a general rule of thumb, only label inside loops in a clockwise direction with circulating currents as the aim is to cover all the elements of the circuit at least once. Any required branch current may be found from the appropriate loop or mesh currents as before using Kirchhoff´s method.

For example: : $i1 = I1$, $i2 = -I2$ and $I3 = I1 – I2$

We now write Kirchhoff's voltage law equation in the same way as before to solve them but the advantage of this method is that it ensures that the information obtained from the circuit equations is the minimum required to solve the circuit as the information is more general and can easily be put into a matrix form

For example, consider the circuit from the previous section.

$$10 = 50I_1 - 40I_2$$
$$-20 = -40I_1 + 60I_2$$
or [V] = [R] [I]



These equations can be solved quite quickly by using a single mesh impedance matrix Z. Each element ON the principal diagonal will be “positive e” and is the total impedance of each mesh. Where as, each element OFF the principal diagonal will either be “zero” or “negative” and represents the circuit element connecting all the appropriate meshes.

First we need to understand that when dealing with matrices, for the division of two matrices it is the same as multiplying one matrix by the inverse of the other as shown.

$$[V]=[I]\times[R] \quad or \quad [R]\times[I]=[V]$$

$$\begin{bmatrix} 50 & -40 \\ -40 & 60 \end{bmatrix} \times \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 10 \\ -20 \end{bmatrix}$$

$$I = \frac{V}{R} = R^{-1} \times V$$

$$Inverse\ of\ R = \begin{bmatrix} 60 & 40 \\ 40 & 50 \end{bmatrix}$$

$$|R| = (60\times50) - (40\times40) = 1400$$

$$\therefore R^{-1} = \frac{1}{1400}\begin{bmatrix} 60 & 40 \\ 40 & 50 \end{bmatrix}$$

having found the inverse of R, as V/R is the same as V x R-1, we can now use it to find the two circulating currents.

$$[I]=\left[R^{-1}\right]\times[V]$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}=\frac{1}{1400}\begin{bmatrix} 60 & 40 \\ 40 & 50 \end{bmatrix}\times\begin{bmatrix} 10 \\ -20 \end{bmatrix}$$

$$I_1=\frac{(60\times10)+(40\times-20)}{1400}=\frac{-200}{1400}=-0.143A$$

$$I_2=\frac{(40\times10)+(50\times-20)}{1400}=\frac{-600}{1400}=-0.429A$$

Where:

(V) gives the total battery voltage for loop 1 and then loop 2

(I) states the names of the loop currents which we are trying to find

(R) is the resistance matrix

(R-1) is the inverse of the [R] matrix

and this gives I1 as -0.143 Amps and I2 as -0.429 Amps

As : I3 = I1 – I2

The combined current of I3 is therefore given as : -0.143 – (-0.429) = 0.286 Amps

This is the same value of 0.286 amps current, we found previously in the Kirchhoffs circuit law tutorial.

Tutorial Summary

This "look-see" method of circuit analysis is probably the best of all the circuit analysis methods with the basic procedure for solving Mesh Current Analysis equations is as follows:

1 .Label all the internal loops with circulating currents. (I_1, I_2, …I_L) etc.

2 .Write the [L x 1] column matrix [V] giving the sum of all voltage sources in each loop.

3.Write the [L x L] matrix, [R] for all the resistances in the circuit as follows:

R_{11} = the total resistance in the first loop.

R_{nn} = the total resistance in the Nth loop.

R_{JK} = the resistance which directly joins loop J to Loop K.

4.Write the matrix or vector equation [V] = [R] x [I] where [I] is the list of currents to be found.

As well as using Mesh Current Analysis, we can also use node analysis to calculate the voltages around the loops, again reducing the amount of mathematics required using just Kirchoff's laws. In the next tutorial relating to DC circuit theory, we will look at Nodal Voltage Analysis to do just that.

How to Use the Mesh Current Method

Diving into the mesh current method, it's important to note that we'll be using the same example circuit (Figure 1) that we've been using to introduce other network analysis methods:

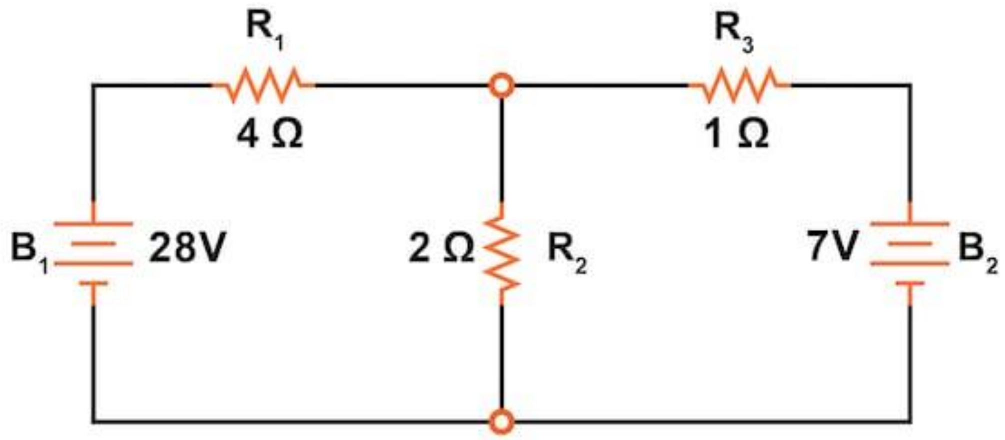


Figure 1. Circuit schematic for explaining the mesh current method.

Step 1: Identify and Label the Current Loops

The first step in the mesh current method is identifying and labeling the current "loops" within the circuit. To do this, we must find at least one loop current passing through every component in the circuit.

In our example circuit (Figure 2), the loop formed by B_1, R_1, and R_2 will be the first loop, while the loop formed by B_2, R_2, and R_3 will be the second.

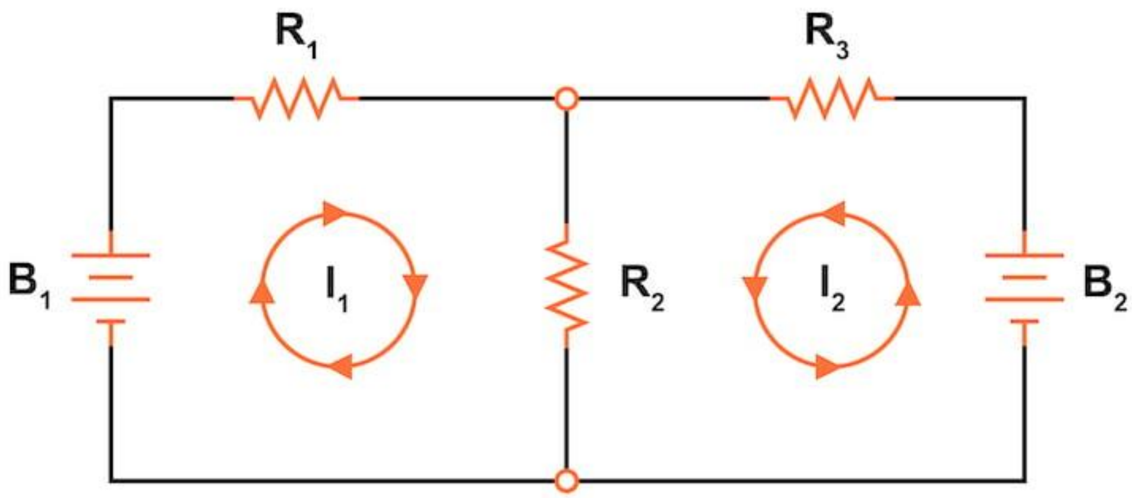


Figure 2. Identify and label the current loops.

The strangest part of the mesh current method is envisioning circulating currents in each of the loops. In fact, this method gets its name from the idea of these currents meshing together between loops like sets of spinning gears.

The choice of each current loop's direction is entirely arbitrary; however, the resulting equations are often easier to solve if the currents are going in the same direction through components with multiple current loops. For example, note how currents I_1 and I_2 both flow "down" through resistor R_2, where they "mesh" or intersect. If the assumed direction of a mesh current is wrong, the answer for that current will have a negative value.

Step 2: Label the Voltage Drop Polarities

The next step is to label all voltage drop polarities across resistors according to the assumed directions of the mesh currents, as shown in Figure 3.

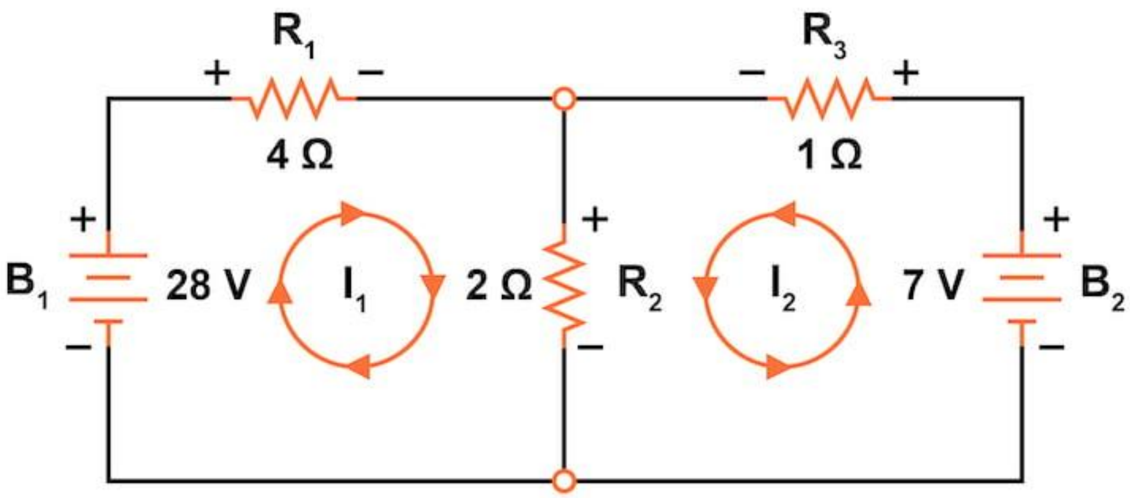


Figure 3. Label the voltage drop polarities.

Remember that the "upstream" end of a resistor will always be positive, and the "downstream" end of a resistor negative. This situation happens because the resistor is a load that drops voltage when current flow through it.

As a note, battery polarities are dictated by their symbol orientations in the diagram and may or may not "agree" with the resistor polarities and assumed loop current directions.

Step 3: Apply Kirchhoff's Voltage Law to Each Loop

Using Kirchhoff's voltage law, we can now step around each of these loops, generating equations representative of the component voltage drops and polarities. As with the branch current method, we will denote a resistor's voltage drop as the product of the resistance (in ohms) and its respective mesh current (that quantity being unknown at this point). Where two currents mesh together, we will write that term in the equation, with the resistor current being the sum of the two meshing currents.

The starting point for tracing the voltage drops around each loop is arbitrary, the same as the direction we trace. Beginning with the left loop of the circuit, let's start at the lower-left corner and trace clockwise, counting polarity as if we had a voltmeter in hand, red lead on the point ahead, and black lead on the point behind. For the left loop with current I_1, we get the following equation:

$$28-R_1I_1-R_2(I_1+I_2)=28-4I_1-2(I_1+I_2)=0$$

Notice that the middle term of the equation uses the sum of mesh currents I_1 and I_2 as the current through resistor R_2. This is because mesh currents I_1 and I_2 are going in the same direction through R_2 and thus complement each other.

We can distribute the coefficient of 2 to the I_1 and I_2 terms and then combine the I_1 terms to simplify the equation as:

$$28-4I_1-2I_1-2I_2=28-6I_1-2I_2=0$$

At this time, we have one equation with two unknowns. To be able to solve for two unknown mesh currents, we must have two equations.

Now let's repeat the process for the right loop of the circuit with current I_2. This will provide us with another KVL equation. With two equations and only two unknown currents, we can solve for the currents. As a creature of habit, we'll again start at the lower-left corner of the right loop and trace clockwise:

$R_2(I_1+I_2)+R_3I_2-7=2(I_1+I_2)+1I_2-7=0$

Simplifying the equation as before, we end up with:

$2I_1+2I_2+1I_2-7=2I_1+3I_2-7=0$

Step 4: Solve the Simultaneous Equations for the Unknown Currents

Now, with two equations, we can use any of several methods to mathematically solve for the unknown currents I_1 and I_2. First, let's rearrange the two equations for an easier solution:

$6I_1+2I_2=28$

$2I_1+3I_2=7$

Now, solving for the currents, we get:

$I_1=5$ A

$I_2 = -1$ A

Step 5: Redraw the Mesh Currents and Determine the Branch Currents

Knowing that these solutions are values for mesh currents, not branch currents, we must go back to our diagram to see how they fit together to give currents through all components (Figure 4).

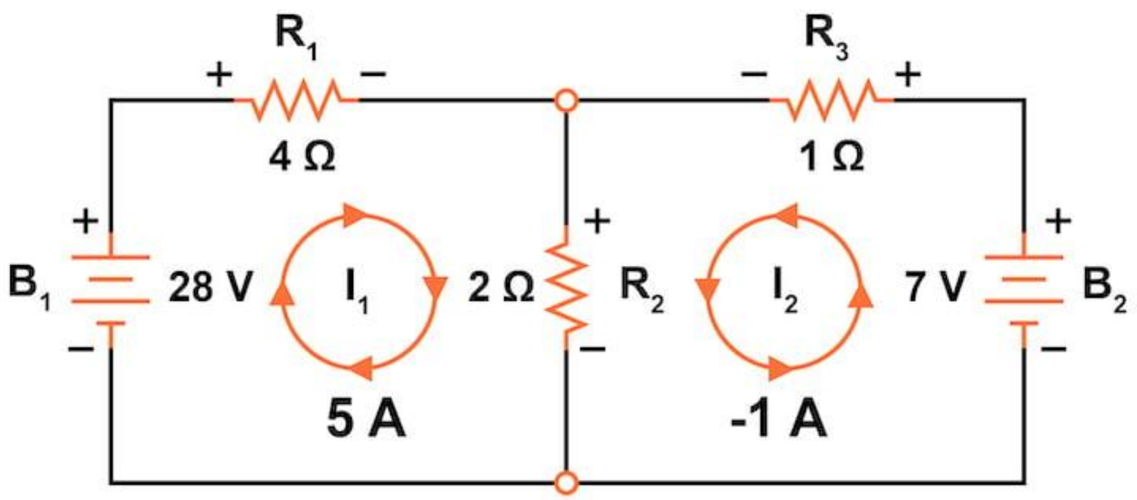


Figure 4. Circuit with calculated mesh current values.

The solution of -1 A for I_2 means that our initially assumed current direction was incorrect. In actuality, I_2 flows in a clockwise direction at a value of +1 A. With this correction, let's redraw our circuit by changing the current flow direction for I_2 and the voltage drop for resistor R_3, as shown in Figure 5.

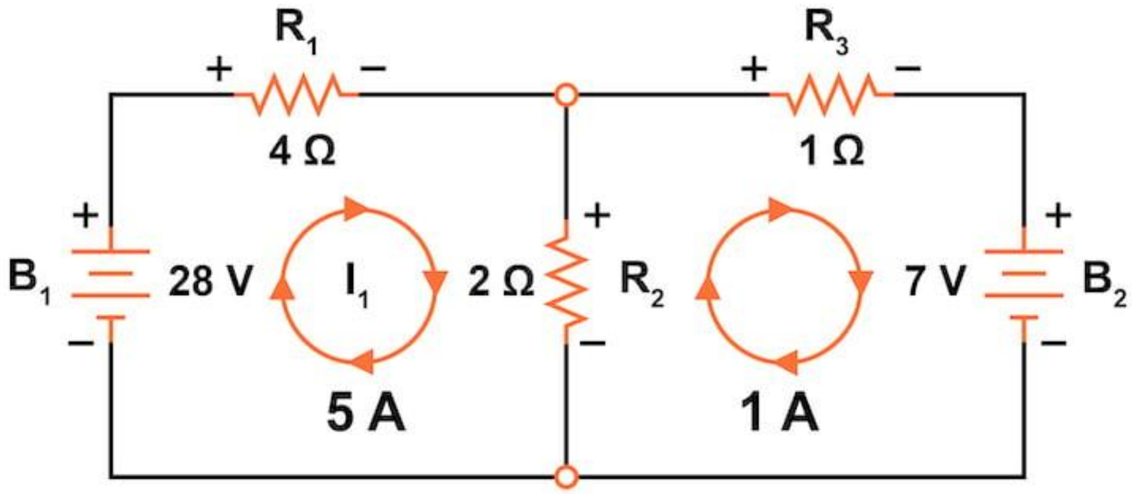


Figure 5. Circuit with corrected mesh current direction for I_2.

Now, from these mesh currents, we can determine the branch currents for our circuit. We can easily see that the current through B_1 and R_1 is 5 A since only mesh current I_1 passes through those two circuit components. Similarly, a current of 1 A is flowing through R_3 and into B_2.

After that, you might question, what about the R_2? It has two mesh currents passing through it. Mesh current I_1 is 5 A going "down" through R_2, while mesh current I_2 is 1 A going "up" through R_2.

To determine the actual current through R_2, we must see how the mesh currents I_1 and I_2 interact. In this case, they're in opposition—flowing in opposite directions. We can algebraically add them to arrive at a final value:

$I\ R_2 = I_1 - I_2 = 5 - 1 = 4$ A

The current through R_2 must be a value of 4 A, going "down." All of the branch currents are shown in Figure 6.

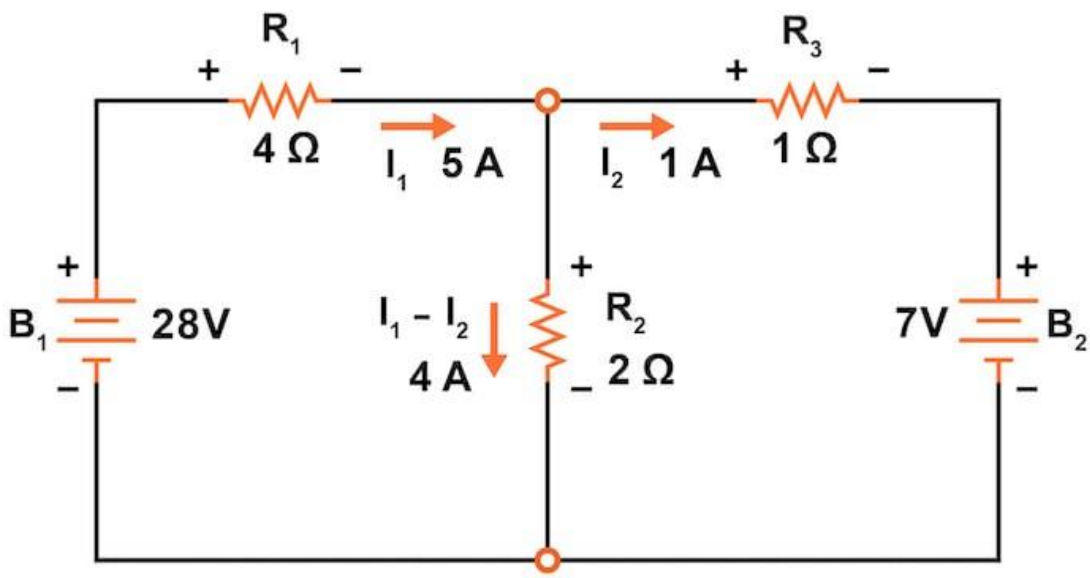


Figure 6. Circuit with calculated branch currents.

Step 6: Calculate the Voltage Drops

Now that we know all of the branch currents, we can use Ohm's law to calculate the unknown voltage drops across the resistors in the circuit.

$V_{R1} = IR_1R_1 = I_1R_1 = 5 \cdot 4 = 20$ V

$V_{R2}=IR_2R_2=(I_1-I_2)R_1=(5-1)\cdot 2=8\ V$

$V_{R3}=IR_3R_3=I_2R_3=1\cdot 1=1\ V$

We could check our results by returning to Kirchhoff's voltage law for our two loops:

$V_{B1}-V_{R1}-V_{R2}=28-20-8=0\ V$

$V_{R2}-V_{R3}-V_{B2}=8-1-7=0\ V$

Advantages of Mesh Current vs Branch Current Methods

The primary advantage of mesh current analysis is that it generally allows for the solution of a large network with fewer unknown values and fewer simultaneous equations. In our example circuit, it requires three equations to solve the branch current method and only two equations using the mesh current method. This advantage can be more pronounced in networks with increased complexity, such as the circuit shown in Figure 7.

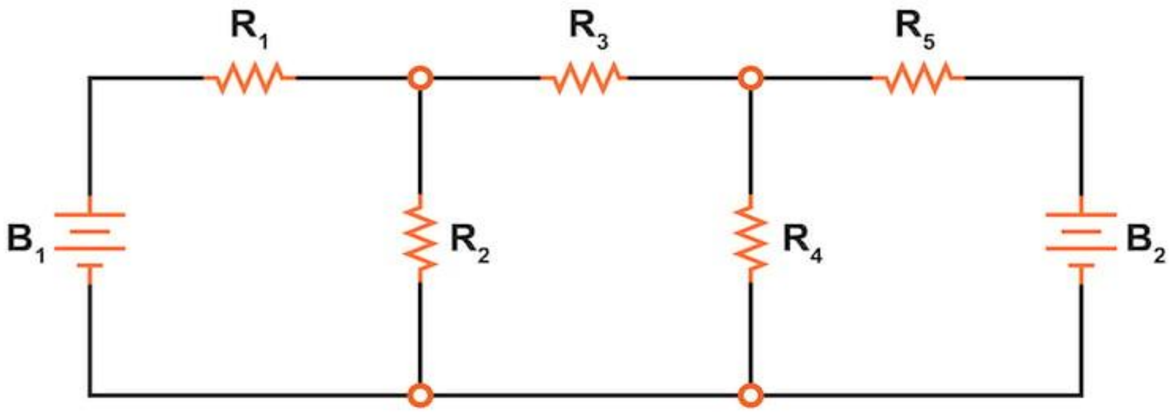


Figure 7. Example circuit with increased complexity.

To solve this network using branch currents, we'd have to establish five variables to account for each and every unique current in the circuit (I_1 through I_5), as demonstrated in Figure 8.

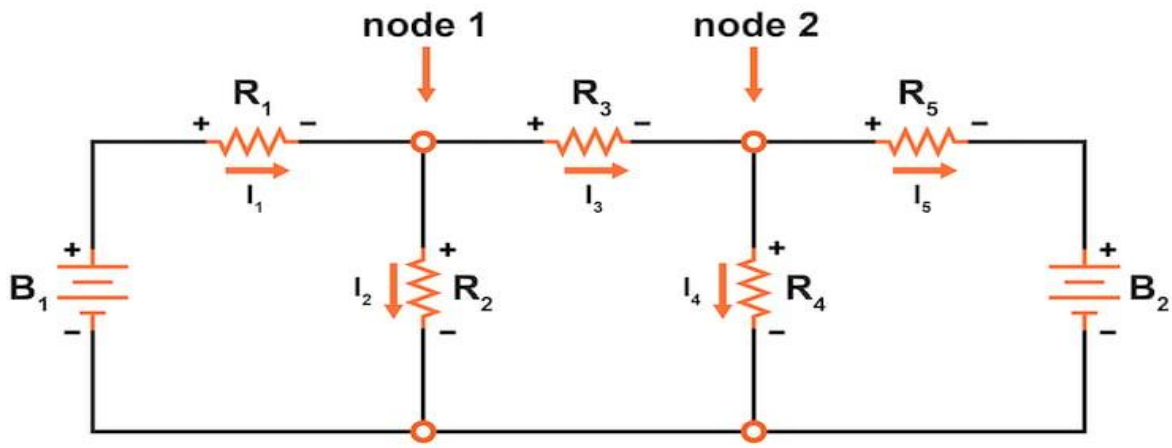


Figure 8. Complex circuit setup for branch current analysis.

This would require five equations for the solution in the form of two KCL equations at the nodes and three KVL equations for the loops:

For the circuit of Figure 8, our five equations would be:

$I_1 - I_2 - I_3 = 0$

KCL at node 1

$I_3 - I_4 - I_5 = 0$

KCL at node 2

$V_{B1} - I_1R_1 - I_2R2 = 0$

KVL of the left loop

$I_2R_2 - I_3R_3 - I_4R_4 = 0$

KVL of the middle loop

$I_4R_4 - I_5R_5 - V_{B2} = 0$

KVL of the right loop

All in all, if you have nothing better to do with your time than to solve for five unknown variables with five equations, you might not mind using the branch current method of analysis for this circuit. However, for those of us who have better things to do with our time, the mesh current method is a whole lot easier, requiring only three unknowns and three equations to solve, as illustrated in Figure 9.

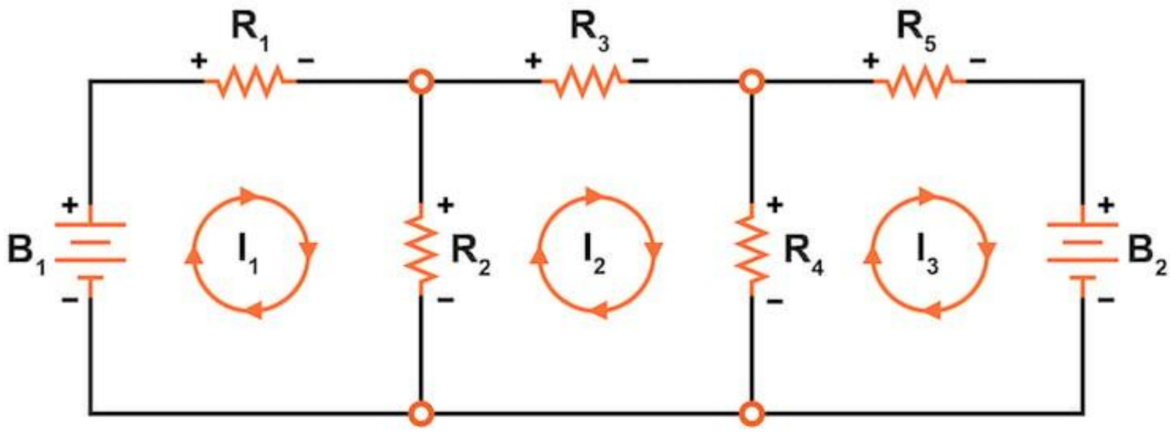


Figure 9. Complex circuit setup for mesh current analysis.

Our three KVL equations using the mesh current method are:

$V_{B1} - I_1R_1 - (I_1 - I_2)R_2 = 0$

KVL of the left loop

$(I_2 - I_1)R_2 - I_2R_3 - (I_2 - I_3)R_4 = 0$

KVL of the middle loop

$(I_3 - I_2)R_4 - I_3R_5 - V_{B2} = 0$

KVL of the right loop

With fewer equations to work with, this method has a clear advantage, especially when performing a simultaneous equation solution by hand without a calculator or computer.

Review of the Mesh Current Method

- The mesh current method is a network analysis technique in which mesh (or loop) currents are assigned arbitrarily, and then Kirchhoff's voltage law and Ohm's law are applied systematically to solve for all unknown currents and voltages.
- **Step 1:** Identify and label the current loops
- **Step 2:** Label the voltage drop polarities
- **Step 3:** Apply Kirchhoff's voltage law to each loop
- **Step 4:** Solve the simultaneous equations for the unknown currents
- **Step 5:** Redraw the mesh currents and determine the branch currents
- **Step 6:** Calculate the voltage drops using Ohm's law
- The mesh current method typically provides an easier solution using fewer simultaneous equations than the branch current method