كلية التقنيات الهندسية

هندسة تقنيات الذكاء الاصطناعي

Lecture: (1)
Vectors

Vectors

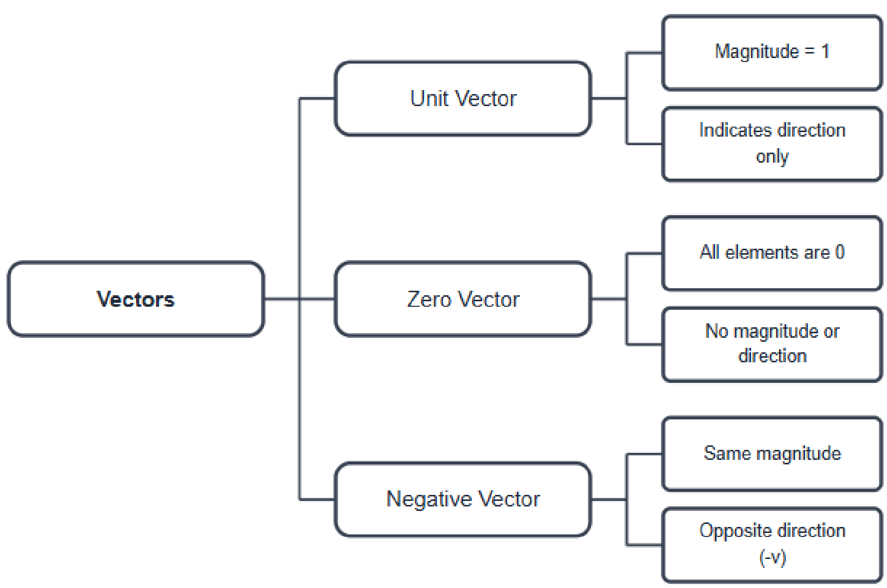
What is a Vector?

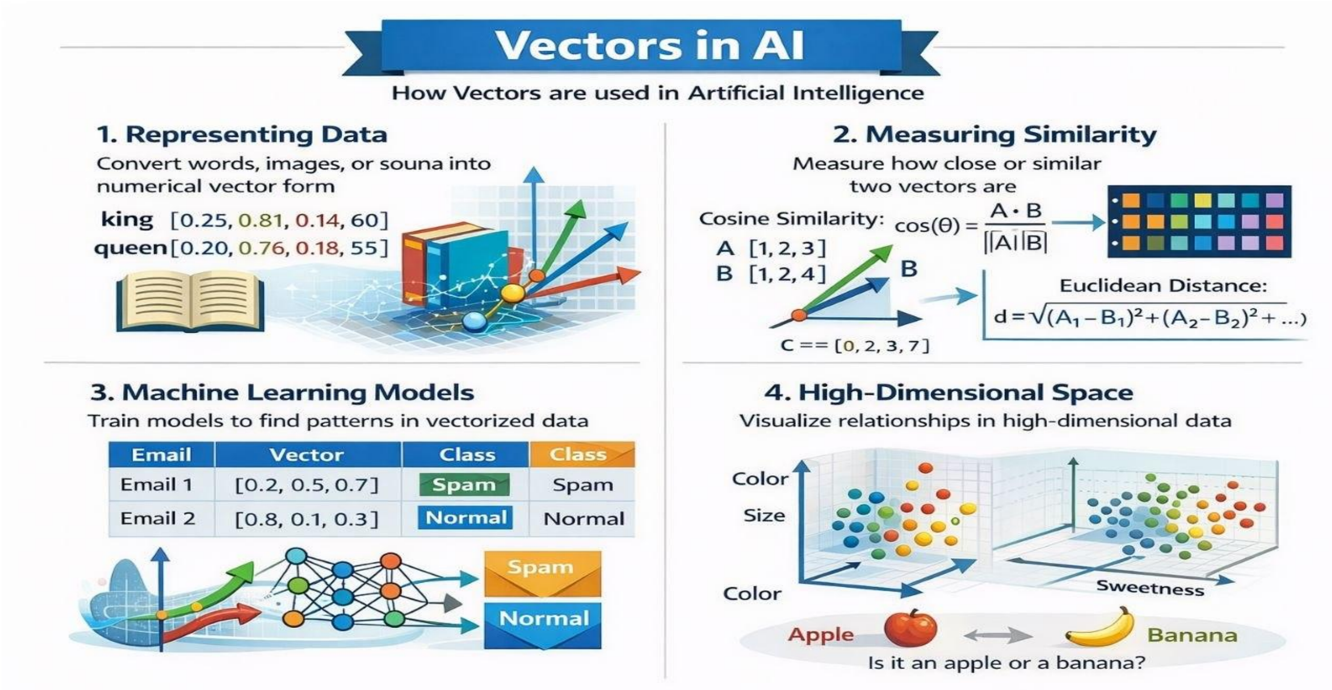
In AI, a vector is an **ordered set** of numbers representing a point in space or a feature set. We typically represent them as **column matrices**:

$$\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$$

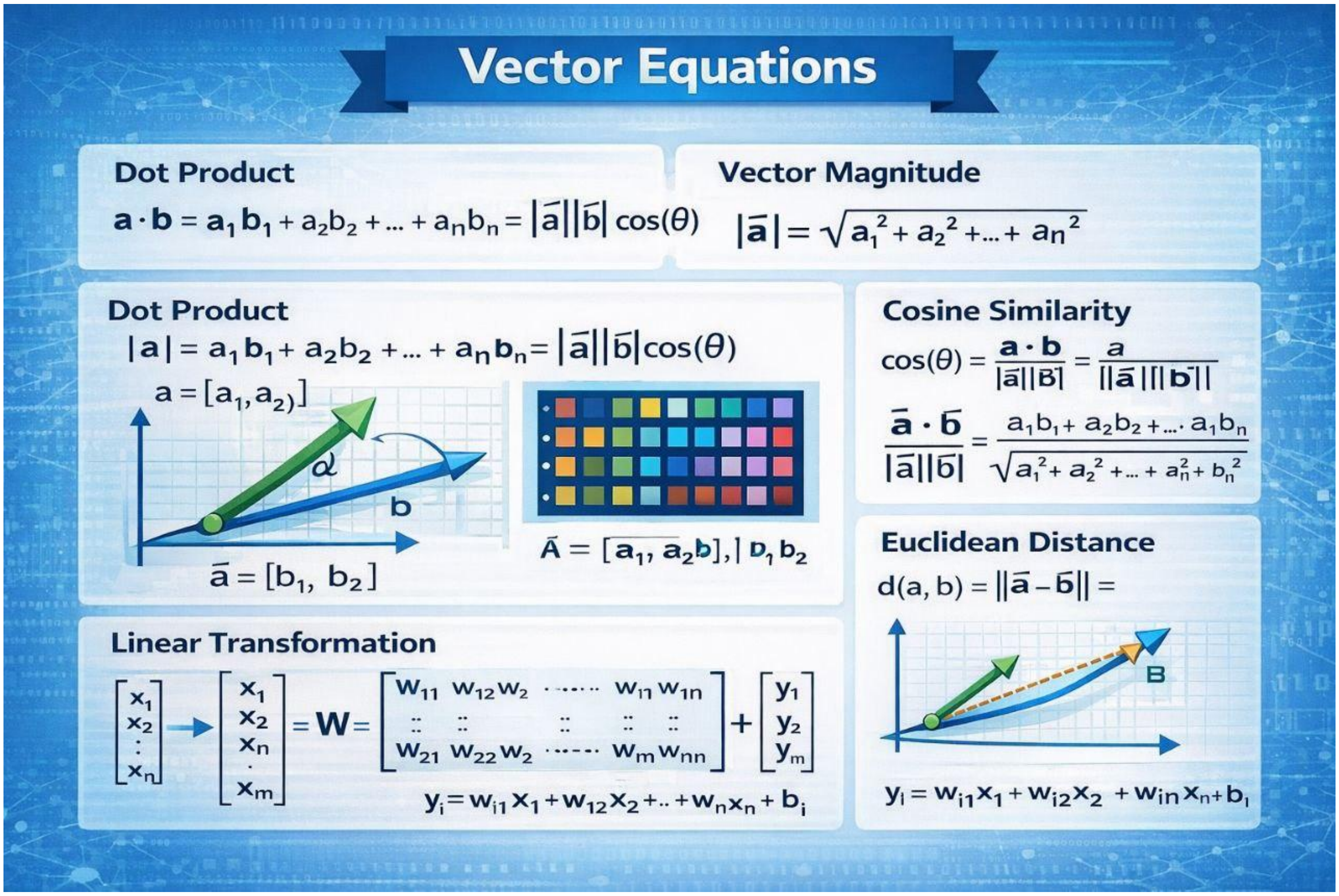
Each element v_i represents a specific dimension or feature (e.g., pixel intensity, word frequency).

Special Vector Types




Vectors in AI
How Vectors are used in Artificial Intelligence
1. Representing Data
Convert words, images, or souna into numerical vector form
king [0.25, 0.81, 0.14, 60]
queen [0.20, 0.76, 0.18, 55]
2. Measuring Similarity
Measure how close or similar two vectors are
Cosine Similarity: cos(θ) = A·B / |A||B|
A [1, 2, 3]
B [1, 2, 4]
B
C == [0, 2, 3, 7]
Euclidean Distance:
d = √(A1 − B1)² + (A2 − B2)² + ...)
3. Machine Learning Models
Train models to find patterns in vectorized data
Email | Vector | Class | Class
Email 1 | [0.2, 0.5, 0.7] | Spam | Spam
Email 2 | [0.8, 0.1, 0.3] | Normal | Normal
Spam
Normal
4. High-Dimensional Space
Visualize relationships in high-dimensional data
Color
Size
Color
Sweetness
Apple
Banana
Is it an apple or a banana?


تطبيقات المتجهات الذكاء الصطناعي
تمثيل النصوص
dog [0.7, 0.6, 0.8]
cat [0.6, 0.7, 0.8]
تمثيل الصور
Image Vector [0.2, 7, 9, 0.1, 1, 8...]
قياس التشابه
Cosine Similarity
Euclidean Distance
A [1, 2, 3]
B [1, 2, 4]
B
تدريب نماذج التعلم الآلي
Spam
Normal
معلجة الصوت
Audio Vector
[0.5, 0.7, 0.3, 0.6]
خوارزيمات البحث والتوصيات
الروية الحاسوبية
Car
Person
Object Detection

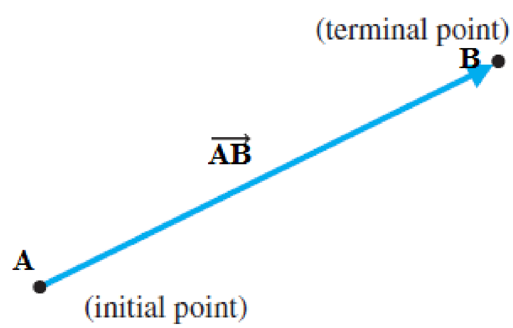


Component Form

A quantity such as force, displacement, or velocity is called a **vector** and is represented by a **directed line segment**.

DEFINITIONS:

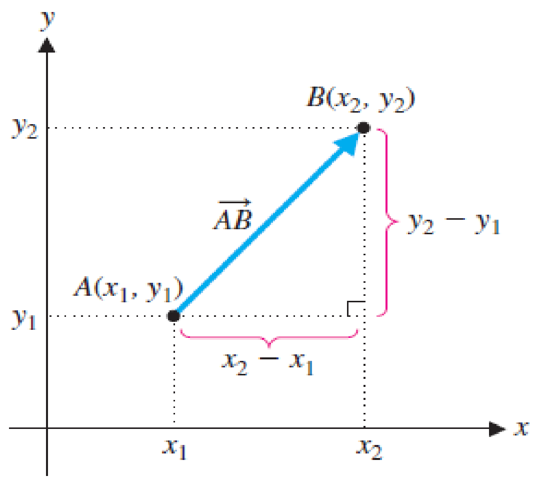
The vector represented by the directed line segment $\overrightarrow{AB}$ has **initial point** *A* and **terminal point** *B*.



Vectors in 2-dimensional coordinates

If ***A*** represent by (x_1, y_1) and

B represent by (x_2, y_2)



Vector is represent by

$\overrightarrow{AB} = \langle v_1, v_2 \rangle$

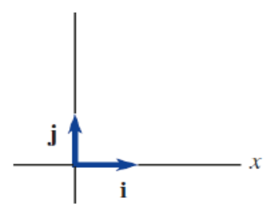
Where $v_1 = x_2 - x_1$

and $v_2 = y_2 - y_1$

or another method to represent the vector

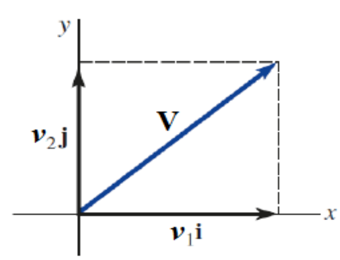
If ***i***, ***j*** are <u>*standard basis vectors*</u>, then

$\mathbf{i} = \langle 1, 0 \rangle$ and $\mathbf{j} = \langle 0, 1 \rangle$,



(a)

$\overrightarrow{AB} = \mathbf{V} = v_1\mathbf{i} + v_2\mathbf{j}.$

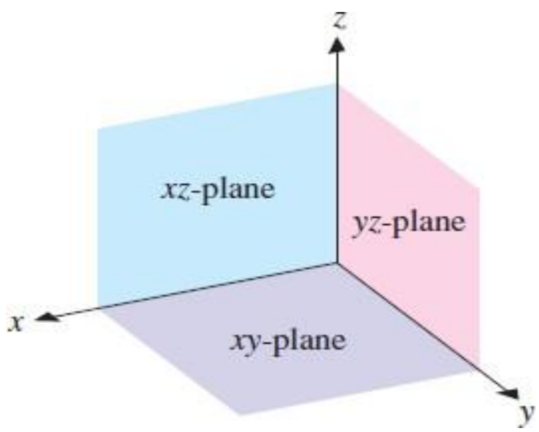


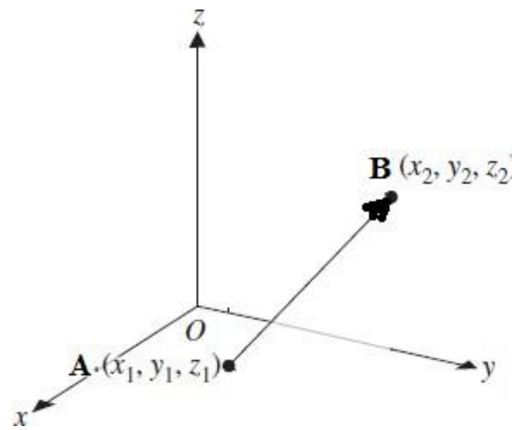
(b)

<u>Note</u>: <u>Zero vector</u>

$V = \langle 0, 0 \rangle$

<u>Vectors in 3-dimensional coordinates (vector in space)</u>





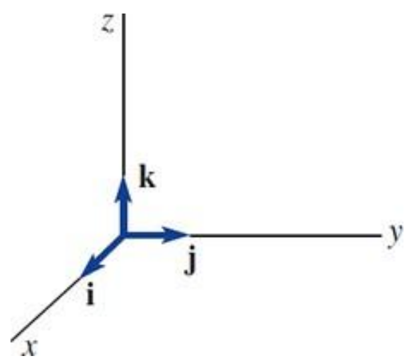
If ***A*** represent by (x_1, y_1, z_1) and B represent by (x_2, y_2, z_2)
Vector is represent by

$\vec{AB} = \langle v_1, v_2, v_3 \rangle$

Where $v_1 = x_2 - x_1$, $v_2 = y_2 - y_1$ *and* $v_3 = z_2 - z_1$

or another method to represent the vector

If ***i*** , ***j, k*** are *<u>standard basis vectors</u>*, then

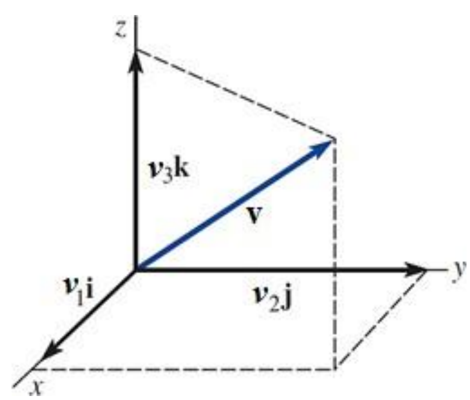


$\mathbf{i} = \langle 1, 0, 0 \rangle,$

$\mathbf{j} = \langle 0, 1, 0 \rangle,$

$\mathbf{k} = \langle 0, 0, 1 \rangle.$

$\vec{AB} = \mathbf{V} = v_1 i + v_2 j + v_3 k$



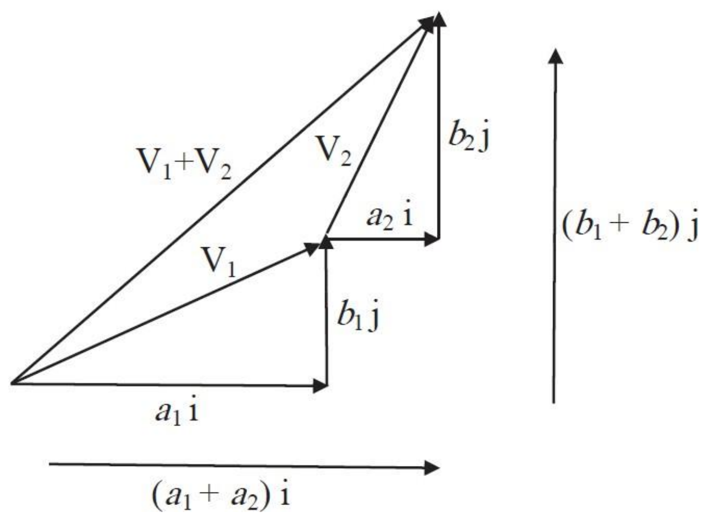
Note*: *Zero vector $V = \langle 0, 0, 0 \rangle$

Algebra of vector:

Algebraic addition:

Let $V_1 = a_1 i + b_1 j$

$V_2 = a_2 i + b_2 j$



Two vector may be added algebraically by adding their corresponding scalar components:

$$V_1 + V_2 = (a_1 + a_2)i + (b_1 + b_2)j$$

Example: If $V_1 = 2i - 5j$ and $V_2 = 4i + 2j$, find $V_1 + V_2$

solution:

$$V_1 + V_2 = (2i - 5j) + (4i + 2j)$$
$$= (2+4)i + (-5+2)j$$
$$= 6i - 3j$$

H.W: The vector $u = 4i + 3j$ and $v = 5i + 6j$, find $u + v$

***Subtraction*:**

Let $V_1 = a_1 i + b_1 j$

$V_2 = a_2 i + b_2 j$

$$V_1 - V_2 = (a_1 - a_2)i + (b_1 - b_2)j$$

***Example*:** If $V_1 = 7i + 3j$ and $V_2 = 2i - 6j$, find $V_1 - V_2$

***solution*:**

$$V_1 - V_2 = (7i + 3j) - (2i - 6j)$$
$$= (7 - 2)i + (3 - (-6))j$$
$$= 5i + 9j$$

***H.W*:** The vector $u = 9i + 6j$ and $v = 5i + 2j$, find $u - v$

***Length of the vector (magnitude)*:**

The length of the vector is $V = ai + bj$ usually denoted by $|V|$, which may be read (the magnitude of V):

$$|V| = |ai + bj| = \sqrt{a^2 + b^2}$$

***Example*:** find length of vector $V = 3i - 5j$

***solution*:**

$$|V| = |3i - 5j| = \sqrt{(3)^2 + (-5)^2} = \sqrt{9 + 25} = \sqrt{34}$$

***H.W*:** The vector $u = 5i + 2j$, find the magnitude (length) of the vector u

Unit vector

Any vector whose length is equal to the unit of length used along the coordinate axes is called a unit vector.

***Direction*:**

$$\text{Direction of the vector } V = \frac{V}{|V|}$$

***Example*:** find the direction of $A = 3i - 4j$

***solution*:**

$$\text{Direction of } A = \frac{A}{|A|} = \frac{3i - 4j}{\sqrt{(3)^2 + (-4)^2}} = \frac{3i - 4j}{\sqrt{25}} = \frac{3}{5}i - \frac{4}{5}j$$

Properties of vectors

Let u, v, w be vector and a, b be scalars

1. $u+v=v+u$
2. $(u+v)+w=u+(v+w)$
3. $u+0=u$
4. $u+(-u)=0$
5. $0u=0$
6. $1u=u$
7. $a(bu)=(ab)u$
8. $a(u+v)=au+av$
9. $(a+b)u=au+bu$

Note : Zero vector $\langle 0,0\rangle$ **or** $\langle 0,0,0\rangle$

Example: For vectors a = $\langle 2,1\rangle$ and b = $\langle 3,-2\rangle$, compute (a) a + b, (b) 2a, (c) 2a + 3b, (d) 2a − 3b and (e) $|2a-3b|$.

Solution:

(a) $\mathbf{a}+\mathbf{b}=\langle 2,1\rangle+\langle 3,-2\rangle=\langle 2+3,1-2\rangle=\langle 5,-1\rangle.$

(b) $2\mathbf{a}=2\langle 2,1\rangle=\langle 2\cdot 2,2\cdot 1\rangle=\langle 4,2\rangle.$

(c) $2\mathbf{a}+3\mathbf{b}=2\langle 2,1\rangle+3\langle 3,-2\rangle=\langle 4,2\rangle+\langle 9,-6\rangle=\langle 13,-4\rangle.$

(d) $2\mathbf{a}-3\mathbf{b}=2\langle 2,1\rangle-3\langle 3,-2\rangle=\langle 4,2\rangle-\langle 9,-6\rangle=\langle -5,8\rangle.$

(e) $|2\mathbf{a}-3\mathbf{b}|=|\langle -5,8\rangle|=\sqrt{25+64}=\sqrt{89}.$